Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 6, No. 2 (2025) 300 –317.

Paper Type: Original Article

Triangular Divergence-Based VIKOR Method Applied in a q-Rung Orthopair Fuzzy Setting for Health Insurance Plan Selection

Mijanur Rahaman Seikh [1,*] **and Arpita Dey** [2]

[1]Department of Mathematics, Kazi Nazrul University, Asansol 713 340, India; mrseikh@ymail.com;
[2]Department of Mathematics, Kazi Nazrul University, Asansol 713340, India; arpita2001d@gmail.com

Citation:

Received: 21-07-2024 Revised: 01-10-2024 Accepted:16-12-2024	Seikh, M. R. & Dey, A. (2025). Triangular Divergence-Based VIKOR Method Applied in a q-Rung Orthopair Fuzzy Setting for Health Insurance Plan Selection. *Journal of Fuzzy Extension and Applications*,6(2), 300-317

Abstract

Selecting the best health insurance plan is a crucial choice, as it directly impacts both financial stability and access to quality healthcare. This process involves weighing several factors, such as coverage, cost, provider network, and customer satisfaction. This manuscript introduces an innovative methodology for evaluating optimal health insurance plans through the adaptation of the "VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR)" technique in conjunction with the "q-Rung Orthopair Fuzzy Set (q-ROFS)" theoretical framework, enhanced by a recently proposed Triangular Divergence Distance Measure (TDDM). This technique provides a complete framework for assessing and rating health insurance policies by fusing the multi-attribute decision-making (MADM) capabilities of the VIKOR method with the flexibility of the q-ROFS theory. We demonstrate how to apply this methodology to the evaluation and ranking of five different health insurance plans according to various criteria through a thorough example. This result shows that plan E from SBI ($\widetilde{\kappa}_5$) is the best health insurance plan. The manuscript further compares this novel methodology with existing MADM techniques and distance measures to illustrate the feasibility of the proposed approach. A sensitivity analysis has been performed to determine its stability. This research advances the development of decision support systems for choosing health insurance plans by providing a strong and adaptable framework that can consider the uncertainties and complexities of making decisions in the real world. Using the VIKOR approach in conjunction with the q-ROFS theory, people and organizations may more adeptly traverse the complex world of health insurance alternatives, resulting in enhanced satisfaction with their selected plans and better outcomes overall.

Keywords: q-rung orthopair fuzzy set; distance measure; multi-attribute decision making; VIKOR; TDDM; health insurance plan evaluation.

Corresponding Author: mrseikh@ymail.com
https://doi.org/10.22105/jfea.2025.455227.1459

1|Introduction

Distance measures play an important role in handling fuzzy information. Several distance measures have been developed for different fuzzy environments over the years. The "q-rung orthopair fuzzy sets (q-ROFSs)" [41] have demonstrated superior ability in handling uncertainty and imprecision inherent in numerous real-world "multiple-attribute decision-making (MADM)" scenarios. They outperform "intuitionistic fuzzy sets (IFSs)" and "Pythagorean fuzzy sets (PFSs)" in this regard. Consequently, several studies concentrating on q-ROFSs have been introduced and utilized for various objectives. Sustainable management and distance measurements for q-ROFSs were recently developed by Cheng et al. [7], who also used the suggested distance measure to the "VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR)" method. Pinar and Boran [29] utilized the q-ROFSs for decision-making problems based on a new distance measure. Du [10] explored generalized orthopair FS and their arithmetic operations. Krishankumar et al. [16] used a "q-rung orthopair fuzzy (q-ROF)" based on an integrated decision-making approach to solving renewable energy source selection problems. Rani and Mishra [30] developed q-ROF for fuel technology. Seikh and Mandal [33] proposed the q-ROF using Frank aggregation operators and its application in MADM with unknown attribute weights. Verma [37] solved a decision-making problem on order-a divergence and entropy measures under q-ROFS. Seikh and Mandal [35] enhanced Archimedean aggregation operators in q-ROFSs and applied for site selection for software operating units. Erdebilli and Sicakyüz [11] used an integrated q-ROF to select the supply-chain management. q-ROFSs have been increasingly employed in various decision-making contexts in recent times [1, 3, 22, 28].

The literature contains a few distance measures for q-ROFSs. The "Euclidean distance measure (EDM)" for "q-rung orthopair fuzzy numbers (q-ROFNs)" was proposed by Biswas et al. [5], who also employed VIKOR to solve certain illustrative MADM problems. Liu et al. [19] used the "Hamming distance measure (HDM)" for q-ROFSs to solve MADM problems. For "Fermatean hesitant fuzzy", Mishra et al. [24] introduced the remoteness index for the MADM method. Additionally, Mandal and Seikh [23] introduced a unique "Evaluation Based on Distance from Average Solution (EDAS)" method for scoring functions to be used in the selection of vacant positions at a company using q-ROFNs. Wang et al. [40] spoke about new q-ROFS distance measures and their uses. Some recent decision-making problems can be found in various fuzzy environments [12, 13, 21, 25, 27, 44]. These distance measures have certain limitations. q-ROFSs, being more advanced and efficient in depicting fuzzy information, automatically call for the development of modified distance measures for better decision-making methods. Recently, the triangular divergence measure, a method generally used in probability distributions, has been researched to develop distance measures based on it. Wang et al. [39] used the triangular divergence measure in MADM for interval-valued fuzzy numbers. Then they utilized the VIKOR method to detect the failure of the machine. Liu [20] developed a distance measure based on triangular divergence for "Fermatean fuzzy sets (FFSs)" applied in medical diagnosis.

Several MADM methods use distance measures to identify the best alternative. One such method is VIKOR. The VIKOR method has been utilized by Cheng et al. [7] in the q-ROF environment to identify sustainable enterprise risk management. The VIKOR method was also used by Cristóbal [31] to solve some illustrative MADM problems. Büyüközkan et al. [6] improved the VIKOR technique for spherical FSs and used it for the selection of renewable energy sources. Akram et al. [2] enhanced the VIKOR method for MADM with complex FFSs. Most of these studies utilized the EDM or the HDM in the VIKOR method. The spherical FSs were proposed by Gündogdu and Kahraman [17], and they were utilized with VIKOR for warehouse site selection. We find instances where the existing distance measures fail to evaluate the distances between q-ROFNs accurately. The triangular divergence measure, proposed by Yehudayoff [42], has been extended to form distance measures by some researchers. In this study, the "triangular divergence-based distance measure (TDDM)" for q-ROFSs is proposed. In order to enhance the current VIKOR approach, we further utilize it.

There are several motivations for this study. They are as follows:

- q-ROFNs have two popular distance measures, but they are not entirely competent in calculating distances between all q-ROFNs. It leads to the requirement of a new and better distance measure for achieving more accurate results in decision-making problems.

- Triangular divergence measure is a popular classical method, mostly used in probability distributions. Distance measurements for FFSs and interval-valued IFSs have been used to it. q-ROFNs are better

at expressing fuzzy information than interval-valued IFSs and FFSs. Hence extending the triangular divergence-based distance measure to q-ROFNs will be more beneficial and realistic.

- The VIKOR methodology, developed by Opricovic [26], represents a valuable and adaptable compromise programming-oriented strategy for addressing MADM problems.

The following are some significant contributions of this study.

- A new triangular divergence-based distance measure is proposed for q-ROFSs and its properties are discussed.
- The VIKOR approach makes use of the suggested distance measure, which ultimately modifies the procedure to improve results for decision-making problems.
- The suggested approach has been used to solve the real-world MADM issue of choosing the best health insurance.

The study has been organized in the following way: Section 2 consists of the preliminaries. Section 3 has the newly proposed distance measure with its properties, and we establish the superiority of the proposed triangular divergence-based distance measure. In Section 4, we iterate the modified-VIKOR method used to solve an illustrative MADM problem. Section 5 consists of the problem definition. Section 6 demonstrates an example of applying our proposed methodology, followed by a comparative analysis and a sensitivity analysis. Lastly, Section 7 has the conclusion, research implications, limitations, and future research scopes.

2|Preliminaries

A few fundamental definitions and introductions are reviewed in this section. The universal set is denoted by Ω throughout this work.

Definition 1. [41] *Consider $\widetilde{\hbar}$ as a q-ROFS defined over Ω is given below*

$$\widetilde{\hbar} = \{\langle \tau, \gamma_{\widetilde{\hbar}}(\tau), \delta_{\widetilde{\hbar}}(\tau)\rangle |\ \tau \in \Omega\}.$$

The mapping $\gamma_{\widetilde{\hbar}} : \Omega \to [0,1]$ represents the membership degree, while the mapping $\delta_{\widetilde{\hbar}} : \Omega \to [0,1]$ signifies the non-membership degree of the element $\tau \in \Omega$ to the set $\widetilde{\hbar}$, subject to the constraint that,

$$0 \leqslant \gamma^q_{\widetilde{\hbar}}(\tau) + \delta^q_{\widetilde{\hbar}}(\tau) \leqslant 1, (q \geqslant 1).$$

Here is the formula for expressing the degree of indeterminacy:

$$\pi_{\widetilde{\hbar}}(\tau) = \sqrt[q]{\gamma^q_{\widetilde{\hbar}}(\tau) + \delta^q_{\widetilde{\hbar}}(\tau) - \gamma^q_{\widetilde{\hbar}}(\tau)\delta^q_{\widetilde{\hbar}}(\tau)}.$$

$P = \langle \gamma_P, \delta_P \rangle$ represents the q-ROFN.

Definition 2. [18] *Let $P = \langle \gamma_P, \delta_P \rangle$, $P_1 = \langle \gamma_{P_1}, \delta_{P_1} \rangle$ and $P_2 = \langle \gamma_{P_2}, \delta_{P_2} \rangle$ be three q-ROFNs. Then,*

- $P^c = \langle \delta_P, \gamma_P \rangle$.
- $P_1 \oplus P_2 = \left\langle \sqrt[q]{\gamma^q_{P_1} + \gamma^q_{P_2} - \gamma^q_{P_1}\gamma^q_{P_2}}, \delta_{P_1}\delta_{P_2} \right\rangle$.
- $P_1 \bigotimes P_2 = \left\langle \gamma_{P_1}\gamma_{P_2}, \sqrt[q]{\delta^q_{P_1} + \delta^q_{P_2} - \delta^q_{P_1}\delta^q_{P_2}} \right\rangle$.
- $\lambda P = \left\langle \sqrt[q]{1-(1-\gamma^q_P)^\lambda}, \delta^\lambda_P \right\rangle, \lambda > 0$.
- $P^\lambda = \left\langle \gamma^\lambda_P, \sqrt[q]{1-(1-\delta^q_P)^\lambda} \right\rangle, \lambda > 0$.

Definition 3. [14] *If $\widetilde{\hbar}_1$ and $\widetilde{\hbar}_2$ be two q-ROFSs. Then,*

- $\widetilde{\hbar}_1 \subseteq \widetilde{\hbar}_2$ *iff* $\forall\ \tau \in \Omega$, $\gamma_{\widetilde{\hbar}_1}(\tau) \leq \gamma_{\widetilde{\hbar}_2}(\tau)$ *and* $\delta_{\widetilde{\hbar}_1}(\tau) \geq \delta_{\widetilde{\hbar}_2}(\tau)$.
- $\widetilde{\hbar}_1 = \widetilde{\hbar}_2$ *iff* $\forall\ \tau \in \Omega$, $\gamma_{\widetilde{\hbar}_1}(\tau) = \gamma_{\widetilde{\hbar}_2}(\tau)$ *and* $\delta_{\widetilde{\hbar}_1}(\tau) = \delta_{\widetilde{\hbar}_2}(\tau)$.

Definition 4. *Suppose P_1 and P_2 be two q-ROFNs, where $P_1 = \langle \gamma_{P_1}, \delta_{P_1} \rangle$ and $P_2 = \langle \gamma_{P_2}, \delta_{P_2} \rangle$. Then, the HDM ($d_H(P_1, P_2)$)* [19] *and the EDM ($d_E(P_1, P_2)$)* [5] *are defined as follows:*

$$d_H(P_1, P_2) = \frac{1}{2}\left(|\gamma^q_{P_1} - \gamma^q_{P_2}| + |\delta^q_{P_1} - \delta^q_{P_2}| + |\pi^q_{P_1} + \pi^q_{P_2}| \right) \tag{1}$$

$$\text{and } d_E(P_1, P_2) = \sqrt{\frac{1}{2}\left[(\gamma^q_{P_1} - \gamma^q_{P_2})^2 + (\delta^q_{P_1} - \delta^q_{P_2})^2 \right]}. \tag{2}$$

Definition 5. [42] *The set $\Phi_n = \{M = (m_1, m_2, ..., m_n) |\ m_i > 0, \sum\limits_{i=1}^{n} m_i = 1\}$, with $n \geq 2$, represents a collection of finite discrete probability distributions. For any M and N belonging to Φ_n, the classical triangular divergence measure between M and N can be expressed as follows:*

$$\Delta(M, N) = \sum_{i=1}^{n} \frac{(m_i - n_i)^2}{m_i + n_i}.$$

Greater triangular divergence indicates greater difference between the probability distributions M and N. Using the above-mentioned equation, the square root of the triangular divergence is given as follows:

$$d(M, N) = \sqrt{\sum_{i=1}^{n} \frac{(m_i - n_i)^2}{m_i + n_i}}$$

where, by convention, $0/0 = 0$.

3|A triangular divergence based modified distance measure for q-ROFSs

In this segment, we introduce an innovative distance measure for q-ROFSs predicated on the triangular divergence measure.

Definition 6. *Let $\widetilde{\hbar}_k = \langle \tau_j, \gamma_{\widetilde{\hbar}_k}(\tau_j), \delta_{\widetilde{\hbar}_k}(\tau_j) \rangle$ for $k = 1, 2$ be two q-ROFSs in $\Omega = \{\tau_1, \tau_2, ..., \tau_m\}$, then the TDDM between q-ROFSs $\widetilde{\hbar}_1$ and $\widetilde{\hbar}_2$, denoted by d_{TV} is given by*

$$d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) \quad = \quad \sqrt{\frac{1}{2m} \sum_{j=1}^{m} \left[\frac{\left(\gamma^q_{\widetilde{\hbar}_1}(\tau_j) - \gamma^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_1}(\tau_j)^q + \gamma^q_{\widetilde{\hbar}_2}(\tau_j)} + \frac{\left(\delta^q_{\widetilde{\hbar}_1}(\tau_j) - \delta^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_1}(\tau_j) + \delta^q_{\widetilde{\hbar}_2}(\tau_j)} \right]}. \tag{3}$$

Theorem **1.** *The distance measure $d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2)$, between the two q-ROFSs $\widetilde{\hbar}_1$ and $\widetilde{\hbar}_2$, follows the following properties. Here $\widetilde{\hbar}_1$, $\widetilde{\hbar}_2$ and $\widetilde{\hbar}_3$ are q-ROFSs.*

I. $d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) = 0 \Leftrightarrow \widetilde{\hbar}_1 = \widetilde{\hbar}_2$;

II. $d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) = d_{TV}(\widetilde{\hbar}_2, \widetilde{\hbar}_1)$;

III. $0 \leq d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) \leq 1$;

IV. *If* $\widetilde{\hbar}_1 \subseteq \widetilde{\hbar}_2 \subseteq \widetilde{\hbar}_3$, *then* $d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) \leq d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_3)$ *and* $d_{TV}(\widetilde{\hbar}_2, \widetilde{\hbar}_3) \leq d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_3)$.

Proof: I. Let $d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) = 0$ for any $\tau_j \in \Omega$. Then,

$$d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) = \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\frac{\left(\gamma^q_{\widetilde{\hbar}_1}(\tau_j) - \gamma^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_1}(\tau_j) + \gamma^q_{\widetilde{\hbar}_2}(\tau_j)} + \frac{\left(\delta^q_{\widetilde{\hbar}_1}(\tau_j) - \delta^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_1}(\tau_j) + \delta^q_{\widetilde{\hbar}_2}(\tau_j)}\right]} = 0.$$

Then we have

$$\frac{\left(\gamma^q_{\widetilde{\hbar}_1}(\tau_j) - \gamma^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_1}(\tau_j) + \gamma^q_{\widetilde{\hbar}_2}(\tau_j)} = \frac{\left(\delta^q_{\widetilde{\hbar}_1}(\tau_j) - \delta^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_1}(\tau_j) + \delta^q_{\widetilde{\hbar}_2}(\tau_j)} = 0.$$

That is,

$$\left(\gamma^q_{\widetilde{\hbar}_1}(\tau_j) - \gamma^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2 = \left(\delta^q_{\widetilde{\hbar}_1}(\tau_j) - \delta^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2 = 0.$$

Again we have,

$$0 \leqslant \gamma_{\widetilde{\hbar}_1},\ \gamma_{\widetilde{\hbar}_2},\ \delta_{\widetilde{\hbar}_1},\ \delta_{\widetilde{\hbar}_2} \leqslant 1.$$

Hence,

$$\gamma_{\widetilde{\hbar}_1}(\tau_j) = \gamma_{\widetilde{\hbar}_2}(\tau_j),\ \delta_{\widetilde{\hbar}_1}(\tau_j) = \delta_{\widetilde{\hbar}_2}(\tau_j).$$

Therefore,

$$\widetilde{\hbar}_1 = \widetilde{\hbar}_2.$$

Sufficiently: When,

$$\widetilde{\hbar}_1 = \widetilde{\hbar}_2.$$

Then,

$$\gamma_{\widetilde{\hbar}_1}(\tau_j) = \gamma_{\widetilde{\hbar}_2}(\tau_j),\ \delta_{\widetilde{\hbar}_1}(\tau_j) = \delta_{\widetilde{\hbar}_2}(\tau_j).$$

Therefore,

$$d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) = \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\frac{\left(\gamma^q_{\widetilde{\hbar}_1}(\tau_j) - \gamma^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_1}(\tau_j) + \gamma^q_{\widetilde{\hbar}_2}(\tau_j)} + \frac{\left(\delta^q_{\widetilde{\hbar}_1}(\tau_j) - \delta^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_1}(\tau_j) + \delta^q_{\widetilde{\hbar}_2}(\tau_j)}\right]} = 0.$$

Hence, property I holds.

II. Next we prove that $d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) = d_{TV}(\widetilde{\hbar}_2, \widetilde{\hbar}_1)$.

$$\begin{aligned} d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) &= \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\frac{\left(\gamma^q_{\widetilde{\hbar}_1}(\tau_j) - \gamma^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_1}(\tau_j) + \gamma^q_{\widetilde{\hbar}_2}(\tau_j)} + \frac{\left(\delta^q_{\widetilde{\hbar}_1}(\tau_j) - \delta^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_1}(\tau_j) + \delta^q_{\widetilde{\hbar}_2}(\tau_j)}\right]} \\ &= \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\frac{\left(\gamma^q_{\widetilde{\hbar}_2}(\tau_j) - \gamma^q_{\widetilde{\hbar}_1}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_2}(\tau_j) + \gamma^q_{\widetilde{\hbar}_1}(\tau_j)} + \frac{\left(\delta^q_{\widetilde{\hbar}_2}(\tau_j) - \delta^q_{\widetilde{\hbar}_1}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_2}(\tau_j) + \delta^q_{\widetilde{\hbar}_1}(\tau_j)}\right]} \\ &= d_{TV}(\widetilde{\hbar}_2, \widetilde{\hbar}_1). \end{aligned}$$

Hence, property II holds.

III. We then prove that $0 \leq d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) \leq 1$.
From Definition 1, it is obvious that $0 \leq d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2)$.
We know that,

$$0 \leq \gamma^q_{\widetilde{\hbar}_1}(\tau) + \gamma^q_{\widetilde{\hbar}_1}(\tau) \leq 1 \text{ and } 0 \leq \ \gamma^q_{\widetilde{\hbar}_2}(\tau) + \gamma^q_{\widetilde{\hbar}_2}(\tau) \leq 1.$$

So, the following inequality hold

$$\left(\gamma^q_{\widetilde{\hbar}_1}(\tau) - \gamma^q_{\widetilde{\hbar}_2}(\tau)\right)^2 \leq \left(\gamma^q_{\widetilde{\hbar}_1}(\tau) + \gamma^q_{\widetilde{\hbar}_2}(\tau)\right)^2$$

$$\text{and } \left(\delta^q_{\widetilde{\hbar}_1}(\tau) - \delta^q_{\widetilde{\hbar}_2}(\tau)\right)^2 \leq \left(\delta^q_{\widetilde{\hbar}_1}(\tau) + \delta^q_{\widetilde{\hbar}_2}(\tau)\right)^2.$$

Then,

$$\begin{aligned}
d_{TV}(\widetilde{\hbar}_1,\widetilde{\hbar}_2) &= \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\frac{\left(\gamma^q_{\widetilde{\hbar}_1}(\tau_j)-\gamma^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_1}(\tau_j)+\gamma^q_{\widetilde{\hbar}_2}(\tau_j)}+\frac{\left(\delta^q_{\widetilde{\hbar}_1}(\tau_j)-\delta^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_1}(\tau_j)+\delta^q_{\widetilde{\hbar}_2}(\tau_j)}\right]} \\
&\le \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\frac{\left(\gamma^q_{\widetilde{\hbar}_1}(\tau_j)+\gamma^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_1}(\tau_j)+\gamma^q_{\widetilde{\hbar}_2}(\tau_j)}+\frac{\left(\delta^q_{\widetilde{\hbar}_1}(\tau_j)+\delta^q_{\widetilde{\hbar}_2}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_1}(\tau_j)+\delta^q_{\widetilde{\hbar}_2}(\tau_j)}\right]} \\
&= \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\gamma^q_{\widetilde{\hbar}_1}(\tau_j)+\gamma^q_{\widetilde{\hbar}_2}(\tau_j)+\delta^q_{\widetilde{\hbar}_1}(\tau_j)+\delta^q_{\widetilde{\hbar}_2}(\tau_j)\right]} \\
&\le \sqrt{\frac{1}{2m}\sum_{j=1}^{m}2} \\
&= 1.
\end{aligned}$$

Hence, property III holds.

IV. Now, we prove that if $\widetilde{\hbar}_1 \subseteq \widetilde{\hbar}_2 \subseteq \widetilde{\hbar}_3$, then $d_{TV}(\widetilde{\hbar}_1,\widetilde{\hbar}_2) \le d_{TV}(\widetilde{\hbar}_1,\widetilde{\hbar}_3)$ and $d_{TV}(\widetilde{\hbar}_2,\widetilde{\hbar}_3) \le d_{TV}(\widetilde{\hbar}_1,\widetilde{\hbar}_3)$.
When, $\widetilde{\hbar}_1 \subseteq \widetilde{\hbar}_2 \subseteq \widetilde{\hbar}_3$, then by Definition 3 we have,

$$\gamma^q_{\widetilde{\hbar}_1} \le \gamma^q_{\widetilde{\hbar}_2} \le \gamma^q_{\widetilde{\hbar}_3},\ \delta^q_{\widetilde{\hbar}_3} \le \delta^q_{\widetilde{\hbar}_2} \le \delta^q_{\widetilde{\hbar}_1}.$$

For $0 \le \eta_k \le 1$ $(k=1,2)$ and $0 \le \eta_1+\eta_2 \le 1$, a function $g(\tau_1,\tau_2)$ could be established as below:

$$g(\tau_1,\tau_2)=\sum_{k=1}^{2}\frac{(\tau_k-\eta_k)^2}{\tau_k+\eta_k},\ \tau_k \in [0,1],$$

then

$$\begin{aligned}
\frac{\partial g}{\partial \tau_k} &= \frac{2(\tau_k-\eta_k)(\tau_k+\eta_k)-(\tau_k-\eta_k)^2}{(\tau_k+\eta_k)^2} \\
&= \frac{(\tau_k-\eta_k)(\tau_k+3\eta_k)}{(\tau_k+\eta_k)^2},\ k=1,2. \qquad (4)
\end{aligned}$$

From the partial derivation function of equation (4),

$$\begin{cases} \frac{\partial g}{\partial \tau_k} \ge 0, & 0 \le \eta_k \le \tau_k \le 1, \\ \frac{\partial g}{\partial \tau_k} < 0, & 0 \le \tau_k < \eta_k \le 1. \end{cases} \qquad (5)$$

Therefore, for $\tau_k \ge \eta_k$ and $\tau_k \le \eta_k$, g is a monotonically increasing and monotonically decreasing function for τ_k respectively.
Let, $\eta_1=\gamma^q_{\widetilde{\hbar}_1}$, $\eta_2=\delta^q_{\widetilde{\hbar}_1}$.
When, $\widetilde{\hbar}_1 \subseteq \widetilde{\hbar}_2 \subseteq \widetilde{\hbar}_3$, we have

$$\eta_1=\gamma^q_{\widetilde{\hbar}_1} \le \gamma^q_{\widetilde{\hbar}_2} \le \gamma^q_{\widetilde{\hbar}_3},\ \delta^q_{\widetilde{\hbar}_3} \le \delta^q_{\widetilde{\hbar}_2} \le \delta^q_{\widetilde{\hbar}_1}=\eta_2.$$

Because $g(\tau_1,\tau_2)$ is monotonically increasing when $\tau_1 \ge \eta_1$ if $\gamma^q_{\widetilde{\hbar}_3} \ge \gamma^q_{\widetilde{\hbar}_2}$, then,

$$g\left(\gamma^q_{\widetilde{\hbar}_3},\delta^q_{\widetilde{\hbar}_3}\right) \ge g\left(\gamma^q_{\widetilde{\hbar}_2},\delta^q_{\widetilde{\hbar}_3}\right). \qquad (6)$$

Meanwhile, because $g(\tau_1,\tau_2)$ is monotonically decreasing when $\tau_2 \le \eta_2$ if $\delta^q_{\widetilde{\hbar}_3} \le \delta^q_{\widetilde{\hbar}_2}$, then,

$$g\left(\gamma^q_{\widetilde{\hbar}_2},\delta^q_{\widetilde{\hbar}_3}\right) \ge g\left(\gamma^q_{\widetilde{\hbar}_2},\delta^q_{\widetilde{\hbar}_2}\right). \qquad (7)$$

Combining equations (6) and (7), we have

$$g\left(\gamma^q_{\widetilde{\hbar}_2}, \delta^q_{\widetilde{\hbar}_2}\right) \leq g\left(\gamma^q_{\widetilde{\hbar}_3}, \delta^q_{\widetilde{\hbar}_3}\right)$$

that is,

$$\frac{\left(\gamma^q_{\widetilde{\hbar}_2} - \gamma^q_{\widetilde{\hbar}_1}\right)^2}{\gamma^q_{\widetilde{\hbar}_2} + \gamma^q_{\widetilde{\hbar}_1}} + \frac{\left(\delta^q_{\widetilde{\hbar}_2} - \delta^q_{\widetilde{\hbar}_1}\right)^2}{\delta^q_{\widetilde{\hbar}_2} + \delta^q_{\widetilde{\hbar}_1}} \leq \frac{\left(\gamma^q_{\widetilde{\hbar}_3} - \gamma^q_{\widetilde{\hbar}_1}\right)^2}{\gamma^q_{\widetilde{\hbar}_3} + \gamma^q_{\widetilde{\hbar}_1}} + \frac{\left(\delta^q_{\widetilde{\hbar}_3} - \delta^q_{\widetilde{\hbar}_1}\right)^2}{\delta^q_{\widetilde{\hbar}_3} + \delta^q_{\widetilde{\hbar}_1}}.$$

Consequently, we have

$$\begin{aligned}
d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) &= \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\frac{\left(\gamma^q_{\widetilde{\hbar}_2}(\tau_j) - \gamma^q_{\widetilde{\hbar}_1}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_2}(\tau_j) + \gamma^q_{\widetilde{\hbar}_1}(\tau_j)} + \frac{\left(\delta^q_{\widetilde{\hbar}_2}(\tau_j) - \delta^q_{\widetilde{\hbar}_1}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_2}(\tau_j) + \delta^q_{\widetilde{\hbar}_1}(\tau_j)}\right]} \\
&\leq \sqrt{\frac{1}{2m}\sum_{j=1}^{m}\left[\frac{\left(\gamma^q_{\widetilde{\hbar}_3}(\tau_j) - \gamma^q_{\widetilde{\hbar}_1}(\tau_j)\right)^2}{\gamma^q_{\widetilde{\hbar}_3}(\tau_j) + \gamma^q_{\widetilde{\hbar}_1}(\tau_j)} + \frac{\left(\delta^q_{\widetilde{\hbar}_3}(\tau_j) - \delta^q_{\widetilde{\hbar}_1}(\tau_j)\right)^2}{\delta^q_{\widetilde{\hbar}_3}(\tau_j) + \delta^q_{\widetilde{\hbar}_1}(\tau_j)}\right]} \\
&= d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_3).
\end{aligned}$$

Hence, $d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_2) \leq d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_3)$ is proved.
Similarly , $d_{TV}(\widetilde{\hbar}_2, \widetilde{\hbar}_3) \leq d_{TV}(\widetilde{\hbar}_1, \widetilde{\hbar}_3)$ could be proved, too.
Hence, property IV holds. □

Next, we utilize the following example to establish the superiority of the proposed TDDM for q-ROFNs.

Example 1. *Let us consider three different sets of q-ROFNS, and each set consists of three different q-ROFNS. For q=4 we have taken three q-ROFNs,* $\widetilde{\hbar}_1 = \langle 0.8199, 0.7821\rangle$, $\widetilde{\hbar}_2 = \langle 0.83, 0.7791\rangle$ *and* $\widetilde{\hbar}_3 = \langle 0.83, 0.7811\rangle$. *For q=3 three q-ROFNs be* $\widetilde{\hbar}'_1 = \langle 0.65, 0.8321\rangle$, $\widetilde{\hbar}'_2 = \langle 0.85, 0.6839\rangle$ *and* $\widetilde{\hbar}'_3 = \langle 0.85, 0.6851\rangle$. *Also, for q=2 three q-ROFNs are* $\widetilde{\hbar}''_1 = \langle 0.485, 0.795\rangle$, $\widetilde{\hbar}''_2 = \langle 0.529, 0.5967\rangle$ *and* $\widetilde{\hbar}''_3 = \langle 0.529, 0.597\rangle$. *In order to demonstrate the superiority of the proposed distance measure, we now compute the EDM ([5]), HDM ([19]), and TDDM (proposed) between two different q-ROFN pairs for different q-values using Equations (2), (1), and (3), respectively.*

Table 1 gives the values of the calculated distances.

Table 1. Comparison of distance measure for q-ROFNs.

q-values	q-ROFN Pair	EDM [5]	HDM [19]	TDDM (proposed)
$q=4$	$(\widetilde{\hbar}_1, \widetilde{\hbar}_2)$	0.016	0.022	**0.017**
	$(\widetilde{\hbar}_1, \widetilde{\hbar}_3)$	0.016	0.022	**0.016**
$q=3$	$(\widetilde{\hbar}'_1, \widetilde{\hbar}'_2)$	0.300	0.339	**0.318**
	$(\widetilde{\hbar}'_1, \widetilde{\hbar}'_3)$	0.300	0.339	**0.317**
$q=2$	$(\widetilde{\hbar}''_1, \widetilde{\hbar}''_2)$	0.197	0.275	**0.201**
	$(\widetilde{\hbar}''_1, \widetilde{\hbar}''_3)$	0.197	0.275	**0.200**

Thus, we see that the EDM and HDM give equal distances for the pairs for different q-values. So these distances are not completely competent for calculating accurate distances between q-ROFNs. However, the proposed TDDM gives different distances for the given pairs for different q-values, thus establishing the superiority of the proposed distance measure.

From Figure 1, we notice that there is a significant difference in the distance measures between the given pairs in the case of TDDM, whereas the existing distance measures fail to distinguish between them.

(a) **(b)**

(c) **ac a**

4|MADM process using modified distance based q-ROF-VIKOR method

Here, we provide a proposed MADM procedure that applies the suggested distance measure for q-ROFNs using a modified VIKOR approach.

Let the set of m alternatives be $\tilde{\kappa} = \{\tilde{\kappa}_1, \tilde{\kappa}_2, ..., \tilde{\kappa}_m\}$ and n criteria such that $\hat{CR} = \{\hat{CR}_1, \hat{CR}_2, ..., \hat{CR}_n\}$ be the set of criteria where their weights are $\varpi_1, \varpi_2, ..., \varpi_n$ respectively. Here $0 \leq \varpi_j \leq 1$ and $\sum\limits_{j=1}^{n} \varpi_j = 1$.

We ask the decision expert (DE) to use linguistic values (LVs) to rate each criterion's relevance in order to determine its weights. Next, the LVs are transformed into the corresponding q-ROFNs. The alternatives are ranked using the triangle distance-based VIKOR method. The following is the method's algorithm:

Step 1: The matrix of LVs established by DE is denoted by $\overline{\mathbb{N}} = \left(\overline{\zeta}_{ij}\right), i = 1, 2, ..., m, j = 1, 2, .., n$. The evaluation of alternative $\tilde{\kappa}_i$ with respect to criterion $\hat{CR}_j$ is represented by $\overline{\zeta}_{ij}$, based on the expert's assessment of the variables in a linguistic format.

Step 2: Distribute the weights among the various criteria.

Step 3: Construct the weighted matrix, $\mathbb{N} = (\zeta_{ij})_{m \times n}$, where,

$$\zeta_{ij} = (\gamma_{ij}, \delta_{ij}) = \left(\sqrt[q]{1 - \left(1 - \gamma_k^q\right)^{\varpi_j}}, \left(\delta_k\right)^{\varpi_j}\right). \tag{8}$$

Step 4: Determine both the "q-ROF-anti-ideal solution (q-ROF-AIS)" and the "q-ROF-ideal solution (q-ROF-IS)". Assume the q-ROF-IS and q-ROF-AIS, which were computed in the following, are represented by the variables ζ_j^+ and ζ_j^-, respectively.

$$\zeta_j^+ = (\gamma_j^+, \delta_j^+) = \begin{cases} (\max \gamma_{ij}, \min \delta_{ij}) & \text{when } \hat{CR}_n \text{ is benefit criteria} \\ (\min \gamma_{ij}, \max \delta_{ij}) & \text{when } \hat{CR}_n \text{ is cost criteria} \end{cases}$$

and

$$\zeta_j^- = (\gamma_j^-, \delta_j^-) = \begin{cases} (\min \gamma_{ij}, \max \delta_{ij}) & \text{when } \hat{CR}_n \text{ is benefit criteria} \\ (\max \gamma_{ij}, \min \delta_{ij}) & \text{when } \hat{CR}_n \text{ is cost criteria.} \end{cases} \quad (9)$$

Step 5: Using the formula (3), find the distance between the weighted decision matrix and q-ROF-IS as well as between q-ROF-IS and q-ROF-AIS.

Step 6: Compute the compromise degree (Q_i), group utility (I_i), and individual regret (S_i) in the following manner:

$$S_i = L_{1,i} = \sum_{j=1}^{n} \varpi_j \frac{d_{TV}(\zeta_j^+, \zeta_{ij})}{d_{TV}(\zeta_j^+, \zeta_j^-)}, \quad (10)$$

$$I_i = L_{\infty,i} = \max_{1 \le j \le n} \left(\varpi_j \frac{d_{TV}(\zeta_j^+, \zeta_{ij})}{d_{TV}(\zeta_j^+, \zeta_j^-)} \right), \quad (11)$$

$$Q_i = \varphi \frac{(S_i - S^+)}{(S^- - S^+)} + (1-\varphi) \frac{(I_i - I^+)}{(I^- - I^+)}. \quad (12)$$

Here $S^+ = \min\limits_i S_i$, $S^- = \max\limits_i S_i$, $I^+ = \min\limits_i I_i$, $I^- = \max\limits_i I_i$ and $\varphi = 0.5$.

Step 7: Next rank the alternatives based on the compromise degree. The lowest compromise degree gives the best alternative.

Figure 2 shows the MADM process utilizing the modified distance-based q-ROF-VIKOR method.

5|Problem definition

We have included the decision to select a health insurance plan in this manuscript because it is a real-life MADM problem. Health insurance is a financial product that provides coverage for medical expenses. By paying regular premiums to an insurance company, policyholders gain access to a range of health services and treatments, which the insurance company helps cover. Choosing a health insurance plan is essential for managing medical costs and ensuring access to necessary healthcare services. It provides financial protection against high medical expenses, reduces the risk of unexpected out-of-pocket costs, and offers peace of mind knowing that you have coverage for routine and emergency care. Additionally, health insurance often includes preventive services and discounts on medications, making it a valuable investment in both short-term health and long-term financial stability.

- **Coverage for medical services:** A health insurance plan's coverage for medical services usually covers a variety of necessary healthcare services, including hospital stays, outpatient treatment, prescription drugs, preventative care, and emergency care.
- **Premium cost:** A health insurance plan's premium cost is the sum that must be paid on a regular basis (often monthly) in order to continue coverage. The kind of plan, the extent of coverage, the insured person's age, and their location are some of the variables that may affect this price.
- **Deductibles and Copayments:** Deductibles are the sums policyholders have to pay out-of-pocket before the insurer starts to pay claims. They are frequently included in health insurance plans. Copayments, often known as copays, are set sums of money that patients must pay when getting specific medical procedures or prescription drugs.

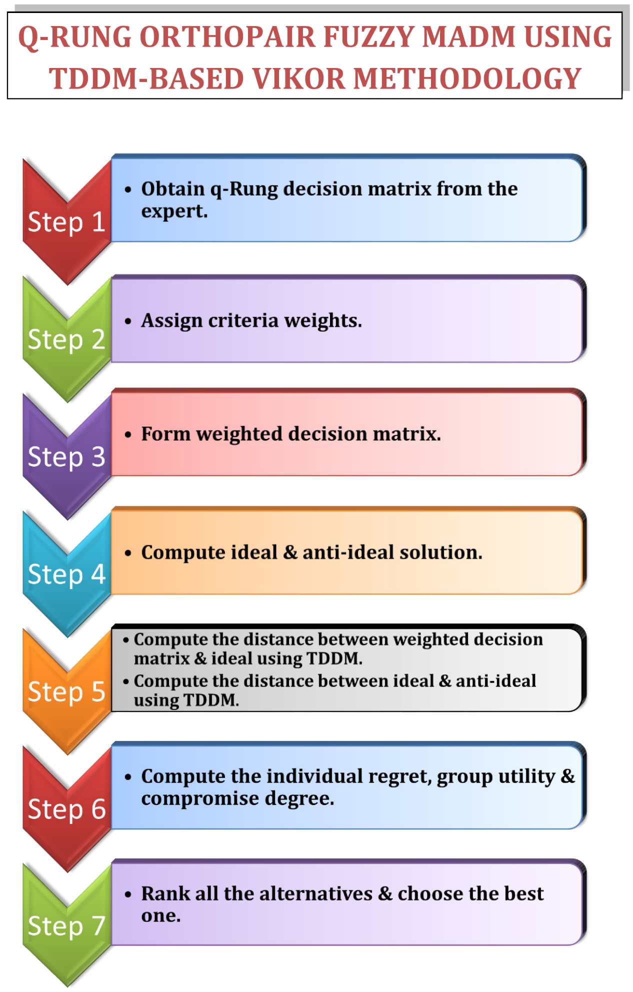

Fig. 2. MADM process using the TDDM-based modi˝ed q-ROF-VIKOR method.

- **In-network provider options:** A network of physicians, specialists, and hospitals that have arranged reduced insurance rates with the insurer makes up the in-network provider options for a health insurance plan. The services provided by these suppliers are more reasonably priced for members who have insurance.
- **Customer service quality:** Customer service quality for health insurance plans is crucial in ensuring that members receive timely and effective support for their healthcare needs. High-quality customer service includes responsive communication, knowledgeable representatives, and efficient claims processing.
- **Policy flexibility and add-ons:** Health insurance plans with add-ons and flexible policies give people the necessary freedom to customize their coverage. Members who choose flexibility can modify provider networks, deductibles, and premiums to better suit their budgetary and medical requirements. Supplementary benefits including dentistry, vision, and wellness programs supplement the base coverage and take care of certain health issues.
- **Coverage for mental health services:** Coverage for mental health services in health insurance plans is crucial for addressing the growing need for mental health care.

- **Network adequacy:** Network adequacy in health insurance plans ensures that there are enough healthcare providers and facilities available to meet the needs of members. This includes evaluating the number of doctors, specialists, and hospitals within a reasonable distance, as well as their availability for timely appointments. Adequate networks are essential for ensuring that members can access necessary care without long wait times or significant travel, ultimately enhancing the overall quality and effectiveness of healthcare services provided under the plan.

6|Illustrative example

This section illustrates the proposed MADM technique by solving a numerical problem. Suppose an insurance company provides a health insurance plan in the state of West Bengal, India. The company has five alternative plans: $\tilde{\kappa}_1$, $\tilde{\kappa}_2$, $\tilde{\kappa}_3$, $\tilde{\kappa}_4$ and $\tilde{\kappa}_5$. There is a DE available, say, D_1. This DE will assess the alternative plans based on the following eight criteria: $\hat{CR}_1$ is the coverage for medical services; $\hat{CR}_2$ is the premium cost; $\hat{CR}_3$ is the deductibles and copayments; $\hat{CR}_4$ is in-network provider options; $\hat{CR}_5$ is the customer service quality; $\hat{CR}_6$ is the policy flexibility and add-ons; $\hat{CR}_7$ is the coverage for mental health services and $\hat{CR}_8$ is the network adequacy. $\hat{CR}_2$ and $\hat{CR}_3$ are "cost criteria", whereas $\hat{CR}_1$, $\hat{CR}_4$, $\hat{CR}_5$, $\hat{CR}_6$, $\hat{CR}_7$ and $\hat{CR}_8$ are "benefit criteria". The framework of the given example is shown in figure 3. We have taken q=4 throughout this paper.

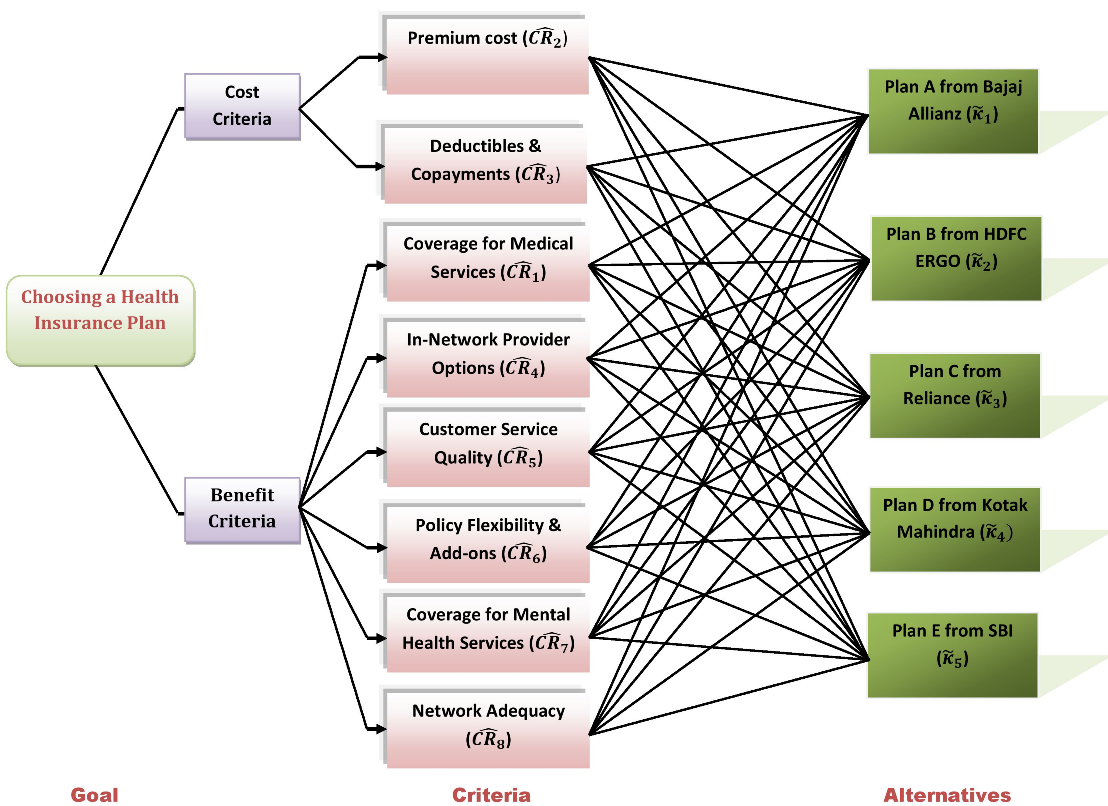

Fig. 3. Framework for choosing health insurance plans.

Table 2 contains the LVs that were used to rank the significance of the DE, criteria, and alternatives.

Next, we randomly assign the criteria's weights. For each of the criteria, $\hat{CR}_1$ through $\hat{CR}_8$, the weights are as follows: $\varpi_1 = 0.15$, $\varpi_2 = 0.22$, $\varpi_3 = 0.18$, $\varpi_4 = 0.02$, $\varpi_5 = 0.12$, $\varpi_6 = 0.08$, $\varpi_7 = 0.15$, and $\varpi_8 = 0.08$, respectively.

The proposed TDDM-based VIKOR approach is then applied to rank the alternatives as the final and final stage.

Table 2. LVs for assessing the importance of DE, criteria, and alternatives.

LVs	Very Good (VG)	Good (G)	Medium (M)	Bad (B)	Very Bad (VB)
q-ROFNs	$\langle 0.98, 0.39\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.85, 0.75\rangle$	$\langle 0.78, 0.84\rangle$	$\langle 0.39, 0.98\rangle$

Step 1 Table 3 provides the decision maker's first decision matrices.

Table 3. Decision matrix for D_1.

Alternative	$\hat{CR}_1$	$\hat{CR}_2$	$\hat{CR}_3$	$\hat{CR}_4$	$\hat{CR}_5$	$\hat{CR}_6$	$\hat{CR}_7$	$\hat{CR}_8$
$\tilde{\kappa}_1$	G	M	VG	G	B	M	G	B
$\tilde{\kappa}_2$	VG	B	M	G	VB	G	B	VG
$\tilde{\kappa}_3$	M	G	VB	G	VG	G	M	B
$\tilde{\kappa}_4$	VB	M	G	VG	M	B	VB	G
$\tilde{\kappa}_5$	B	VB	M	B	VG	G	VG	B

Step 2 The decision matrix is presented in Table 4.

Table 4. Decision matrix.

Alternative	$\hat{CR}_1$	$\hat{CR}_2$	$\hat{CR}_3$	$\hat{CR}_4$	$\hat{CR}_5$	$\hat{CR}_6$	$\hat{CR}_7$	$\hat{CR}_8$
$\tilde{\kappa}_1$	$\langle 0.9, 0.7\rangle$	$\langle 0.85, 0.75\rangle$	$\langle 0.98, 0.39\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.78, 0.84\rangle$	$\langle 0.85, 0.75\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.78, 0.84\rangle$
$\tilde{\kappa}_2$	$\langle 0.98, 0.39\rangle$	$\langle 0.78, 0.84\rangle$	$\langle 0.85, 0.75\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.39, 0.98\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.78, 0.84\rangle$	$\langle 0.98, 0.39\rangle$
$\tilde{\kappa}_3$	$\langle 0.85, 0.75\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.39, 0.98\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.98, 0.39\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.85, 0.75\rangle$	$\langle 0.78, 0.84\rangle$
$\tilde{\kappa}_4$	$\langle 0.39, 0.98\rangle$	$\langle 0.85, 0.75\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.98, 0.39\rangle$	$\langle 0.85, 0.75\rangle$	$\langle 0.78, 0.84\rangle$	$\langle 0.39, 0.98\rangle$	$\langle 0.9, 0.7\rangle$
$\tilde{\kappa}_5$	$\langle 0.78, 0.84\rangle$	$\langle 0.39, 0.98\rangle$	$\langle 0.85, 0.75\rangle$	$\langle 0.78, 0.84\rangle$	$\langle 0.98, 0.39\rangle$	$\langle 0.9, 0.7\rangle$	$\langle 0.98, 0.39\rangle$	$\langle 0.78, 0.84\rangle$

Step 3 We calculate the weighted decision matrix using Equation (8) given in Table 5.

Table 5. Weighted decision matrix.

Alternative	$\hat{CR}_1$	$\hat{CR}_2$	$\hat{CR}_3$	$\hat{CR}_4$	$\hat{CR}_5$	$\hat{CR}_6$	$\hat{CR}_7$	$\hat{CR}_8$
$\tilde{\kappa}_1$	$\langle 0.620, 0.948\rangle$	$\langle 0.622, 0.939\rangle$	$\langle 0.779, 0.844\rangle$	$\langle 0.381, 0.992\rangle$	$\langle 0.482, 0.979\rangle$	$\langle 0.489, 0.977\rangle$	$\langle 0.620, 0.948\rangle$	$\langle 0.436, 0.986\rangle$
$\tilde{\kappa}_2$	$\langle 0.751, 0.868\rangle$	$\langle 0.558, 0.962\rangle$	$\langle 0.594, 0.949\rangle$	$\langle 0.381, 0.993\rangle$	$\langle 0.230, 0.998\rangle$	$\langle 0.535, 0.972\rangle$	$\langle 0.509, 0.974\rangle$	$\langle 0.656, 0.927\rangle$
$\tilde{\kappa}_3$	$\langle 0.569, 0.958\rangle$	$\langle 0.676, 0.924\rangle$	$\langle 0.255, 0.996\rangle$	$\langle 0.381, 0.992\rangle$	$\langle 0.717, 0.893\rangle$	$\langle 0.534, 0.972\rangle$	$\langle 0.569, 0.958\rangle$	$\langle 0.436, 0.986\rangle$
$\tilde{\kappa}_4$	$\langle 0.243, 0.997\rangle$	$\langle 0.622, 0.939\rangle$	$\langle 0.647, 0.938\rangle$	$\langle 0.472, 0.981\rangle$	$\langle 0.540, 0.966\rangle$	$\langle 0.436, 0.986\rangle$	$\langle 0.243, 0.997\rangle$	$\langle 0.535, 0.972\rangle$
$\tilde{\kappa}_5$	$\langle 0.509, 0.974\rangle$	$\langle 0.268, 0.996\rangle$	$\langle 0.594, 0.949\rangle$	$\langle 0.310, 0.996\rangle$	$\langle 0.717, 0.893\rangle$	$\langle 0.535, 0.972\rangle$	$\langle 0.751, 0.868\rangle$	$\langle 0.436, 0.986\rangle$

Step 4 Next, we determine q-ROF-AIS and q-ROF-IS. Table 6 contains the results of our calculation of the q-ROF-IS and q-ROF-AIS using Equation (9).

Table 6. Ideal and anti-ideal solution.

Alternative	$\hat{CR}_1$	$\hat{CR}_2$	$\hat{CR}_3$	$\hat{CR}_4$	$\hat{CR}_5$	$\hat{CR}_6$	$\hat{CR}_7$	$\hat{CR}_8$
ζ_j^+	$\langle 0.751, 0.868\rangle$	$\langle 0.268, 0.996\rangle$	$\langle 0.255, 0.996\rangle$	$\langle 0.472, 0.981\rangle$	$\langle 0.717, 0.893\rangle$	$\langle 0.535, 0.972\rangle$	$\langle 0.751, 0.868\rangle$	$\langle 0.656, 0.927\rangle$
ζ_j^-	$\langle 0.243, 0.997\rangle$	$\langle 0.676, 0.924\rangle$	$\langle 0.779, 0.844\rangle$	$\langle 0.310, 0.997\rangle$	$\langle 0.230, 0.998\rangle$	$\langle 0.436, 0.986\rangle$	$\langle 0.243, 0.997\rangle$	$\langle 0.436, 0.986\rangle$

Step 5 Then, using the proposed TDDM provided in Equation (3), we determine each alternative's distance from the q-ROF-IS as well as between the q-ROF-IS and q-ROF-AIS.

Table 7. Individual regret, group utility, compromise degree and rank.

Alternative	S_i	I_i	Q_i	Rank
$\tilde{\kappa}_1$	0.734	0.182	0.759	4
$\tilde{\kappa}_2$	0.474	0.138	0.288	2
$\tilde{\kappa}_3$	0.498	0.22	0.696	3
$\tilde{\kappa}_4$	0.799	0.182	0.825	5
$\tilde{\kappa}_5$	0.304	0.113	0	1

Step 6 Using Equations (10), (11), and (12), we finally compute individual regret, compromise degree, and group utility. Table 7 provides these values as well as the final ranking.

Step 7 We rank the alternatives according to compromise degrees and observe that the 5^{th} alternative $\tilde{\kappa}_5$, is the best choice, followed by $\tilde{\kappa}_2$, $\tilde{\kappa}_3$, $\tilde{\kappa}_1$ and $\tilde{\kappa}_4$.

6.1|Comparative analysis

This section presents a comparison study of the current techniques and distance metrics with the suggested improved VIKOR methodology. First, we compare the TDDM-based VIKOR method with EDM-based VIKOR, which is given in Equation (2), and also with HDM-based VIKOR, which is given in Equation (1). The comparison is performed by using TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution) [4] in place of the proposed modified VIKOR method. We compare the TDDM-based TOPSIS approach with EDM-based TOPSIS and HDM-based TOPSIS to evaluate the proximity between the options and the q-ROF-IS as well as q-ROF-AIS. Lastly, a comparison is done with the compromise ranking of alternatives from distance to ideal solution (CRADIS) [43]. For comparison, the CRADIS is used in place of the proposed modified VIKOR method. Illustrated in Table 8 are the distances of alternatives from q-ROF-IS and q-ROF-AIS, the proximity coefficient (pertaining to TOPSIS), the level of concession (for VIKOR and CRADIS) for each option, and their respective rankings utilizing varied proximity metrics. Consequently, it can be argued that the suggested TDDM-based VIKOR technique excels in both performance and dependability.

Table 8. Comparison of ranking using di˙erent distance-based MADM methods.

MADM method	Ranking of alternatives based on	Distance measure	Ranking result
TOPSIS [4]	closeness coefficient	TDDM	$\tilde{\kappa}_5 > \tilde{\kappa}_3 > \tilde{\kappa}_2 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
		EDM	$\tilde{\kappa}_5 > \tilde{\kappa}_3 > \tilde{\kappa}_2 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
		HDM	$\tilde{\kappa}_5 > \tilde{\kappa}_3 > \tilde{\kappa}_2 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
CRADIS [43]	compromise degree	TDDM	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
		EDM	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_4 > \tilde{\kappa}_1$
		HDM	$\tilde{\kappa}_2 > \tilde{\kappa}_5 > \tilde{\kappa}_3 > \tilde{\kappa}_4 > \tilde{\kappa}_1$
VIKOR (proposed)	compromise degree	TDDM	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
		EDM	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_4 > \tilde{\kappa}_1$
		HDM	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_4 > \tilde{\kappa}_1$

The data from Table 8 reveals that the hierarchy of TDDM, EDM, and HDM-based TOPSIS remains consistent, i.e., $\tilde{\kappa}_5 > \tilde{\kappa}_3 > \tilde{\kappa}_2 > \tilde{\kappa}_1 > \tilde{\kappa}_4$. The ranking order of TDDM, EDM, and HDM-based CRADIS is $\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$, $\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_4 > \tilde{\kappa}_1$, and $\tilde{\kappa}_2 > \tilde{\kappa}_5 > \tilde{\kappa}_3 > \tilde{\kappa}_4 > \tilde{\kappa}_1$, respectively. The ranking order of TDDM, EDM, and HDM-based VIKOR is $\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$, $\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_4 > \tilde{\kappa}_1$, and $\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_4 > \tilde{\kappa}_1$, respectively. Therefore, we see that the best alternative is $\tilde{\kappa}_5$ for all cases except in HDM-based CRADIS. Hence, it can be asserted that the modified VIKOR method proposed by us holds practical applicability. Figure 4 shows the comparative analysis concerning distance.

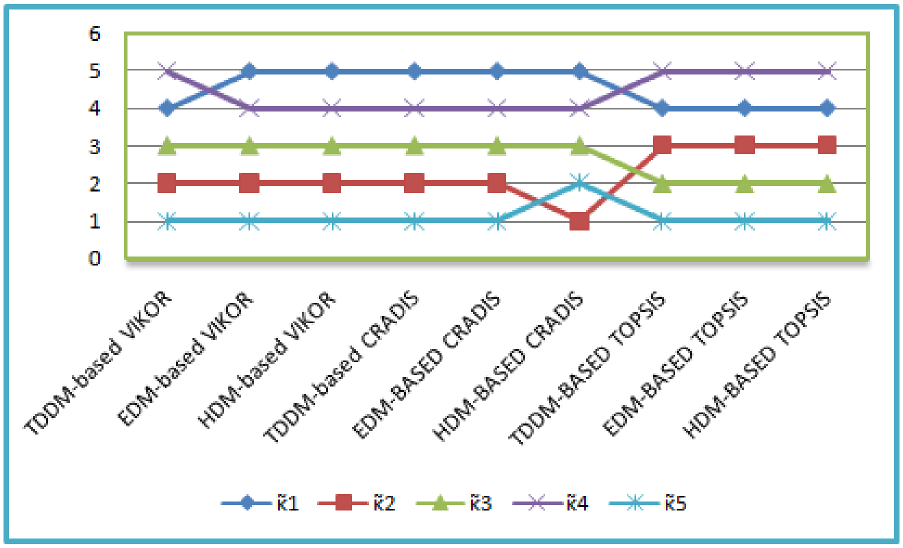

Fig. 4. Comparative analysis.

6.2|Sensitivity analysis

A sensitivity analysis is carried out in this subsection to determine how flexible the proposed approach is. We have employed eight criteria within the context of the case study. To conduct the sensitivity analysis, six sets of criterion weights are utilized, which are derived from reorganizing the initially calculated criterion weights. Through evaluating the model's reaction to various weighting methodologies, individuals can pinpoint the factors that carry the most substantial influence on the outcomes and recognize potential origins of ambiguity. The six sets of criterion weights are presented in Table 9. The proposed approach utilizes six sets of criterion weights, leading to variations in the hierarchical arrangement of the alternative options.

Table 9. Sets of weights of criteria.

Set	$\hat{CR}_1$	$\hat{CR}_2$	$\hat{CR}_3$	$\hat{CR}_4$	$\hat{CR}_5$	$\hat{CR}_6$	$\hat{CR}_7$	$\hat{CR}_8$
I	0.15	0.22	0.18	0.02	0.12	0.08	0.15	0.08
II	0.22	0.18	0.02	0.12	0.08	0.15	0.08	0.15
III	0.18	0.02	0.12	0.08	0.15	0.08	0.15	0.22
IV	0.02	0.12	0.08	0.15	0.08	0.15	0.22	0.18
V	0.12	0.08	0.15	0.08	0.15	0.22	0.18	0.02
VI	0.08	0.15	0.08	0.15	0.22	0.18	0.02	0.12

It is clear from the data in Table 10 that the ranking order of the alternatives is constant for each of the six sets of criteria weights. It is evident that in all instances, plan E from SBI ($\tilde{\kappa}_5$) consistently secures the top position, succeeded by plan B from HDFC ERGO ($\tilde{\kappa}_2$), plan C from Reliance ($\tilde{\kappa}_3$), and plan A from Bajaj Allianz ($\tilde{\kappa}_1$). Ultimately, plan D from Kotak Mahindra ($\tilde{\kappa}_4$) occupies the final position. The utility index of every alternative has been defined, which corresponds to the six sets of criteria weights as shown in Table 10. The compromise degree of all alternatives remains notably close together in all sets. As a result, the ranking results' consistency suggests that our suggested technique is very stable and effective for a variety of criterion weight combinations. Figure 5 provides a visual representation of sensitivity analysis.

7|Conclusion

This research suggests a modified version of the VIKOR method utilizing a triangular divergence distance measure within the q-ROF setting. The q-ROFNs exhibit higher efficacy in handling fuzzy data in comparison to fuzzy extensions. A triangular divergence-based distance measure is suggested in light of the drawbacks of the

Table 10. Sensitivity of proposed method.

Weight set	Q_1	Q_2	Q_3	Q_4	Q_5	Ranking
I	0.759	0.288	0.696	0.825	0	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
II	0.754	0.293	0.694	0.825	0	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
III	0.750	0.294	0.697	0.816	0	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
IV	0.747	0.289	0.695	0.817	0	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
V	0.750	0.288	0.697	0.817	0	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$
VI	0.753	0.287	0.700	0.820	0	$\tilde{\kappa}_5 > \tilde{\kappa}_2 > \tilde{\kappa}_3 > \tilde{\kappa}_1 > \tilde{\kappa}_4$

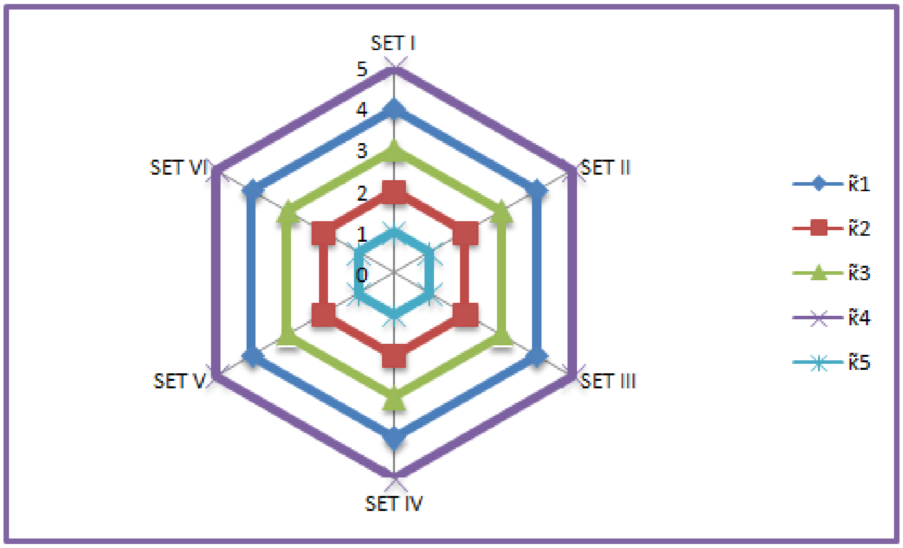

Fig. 5. An analysis of sensitivity.

current q-ROFN distance measures. The superiority of the proposed TDDM is established by an example where the proposed TDDM successfully distinguishes the distances between two given pairs of q-ROFNs. To validate the proposed modified VIKOR method's practical applicability, it is used to solve a numerical problem. The suggested modified VIKOR approach has been examined with other methodologies and distance measurements to determine its applicability. The comparison study indicates that the suggested technique is reliable and better. Sensitivity analysis is performed to verify that the suggested improved VIKOR technique is stable. It follows that the suggested approach provides a number of advantages.

Every research endeavor inevitably encounters certain constraints. In this paper, only the VIKOR approach is modified. Modifying other distance-based MADM methods using TDDM can establish the practicality of the proposed distance measure. Also, only one numerical problem is solved using the proposed modified VIKOR method. Additionally, we have consistently employed a value of q=4 throughout this manuscript. Modifying the value of q may result in superior outcomes. There has just been one decision expert engaged. Better outcomes can be achieved by involving more decision specialists in the decision-making process. Another limitation is that in order to keep the method's simplicity, we have assumed the weight of the criteria. Diverse criterion weight determination techniques can greatly enhance the model.

There are a lot of directions that might be explored further. The proposed distance measure can be extended to other fuzzy environments such as interval-valued FFSs [15], p, q-quasirung orthopair FSs [34], hesitant FSs [36], neutrosophic FSs [9], 3,4-quasirung FSs [32], 2-tuple linguistic PFSs [38] and interval-valued spherical FSs [8]. Furthermore, various distance-based MADM techniques can be modified utilizing the suggested TDDM. The suggested TDDM-based modified VIKOR can be used to further demonstrate the method's usefulness by applying it to other real-world MADM situations.

Acknowledgments

The authors would like to convey their profound appreciation to the editors and anonymous reviewers for their invaluable insights and constructive comments, which have substantially facilitated the improvement of this manuscript.

Author Contribution

M.R. Seikh: conceptualization, reviewing and editing. A. Dey: methodology, writing original graft, software. All authors have read and agreed to the published version of the manuscript.

Funding

The authors declare that no external funding or support was received for the research presented in this paper, including administrative, technical, or in-kind contributions.

Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare that there is no conflict of interest.

References

[1] Akram, M., Bashir, A., & Edalatpanah, S. A. (2021). A hybrid decision-making analysis under complex q-rung picture fuzzy Einstein averaging operators.*Computational and Applied Mathematics*, 40, 1–35. https://doi.org/10.1007/s40314-021-01651-y

[2] Akram, M., Muhiuddin, G., & Santos-García, G. (2022). An enhanced VIKOR method for multi-criteria group decision-making with complex Fermatean fuzzy sets. *Mathematical Biosciences and Engineering*, 19(7), 7201–7231. 10.3934/mbe.2022340

[3] Akram, M., Naz, S., Edalatpanah, S. A., & Samreen, S. (2023). A hybrid decision-making framework under 2-tuple linguistic complex q-rung orthopair fuzzy Hamy mean aggregation operators.*Computational and Applied Mathematics*, 42(3), 118. https://doi.org/10.1007/s40314-023-02254-5

[4] Alkan, N., & Kahraman, C. (2021). Evaluation of government strategies against COVID-19 pandemic using q-rung orthopair fuzzy TOPSIS method. *Applied Soft Computing,* 110, 107653. https://doi.org/10.1016/j.asoc.2021.107653

[5] Biswas, S., Pamucar, D., Raj, A., & Kar, S. (2023). A proposed q-rung orthopair fuzzy-based decision support system for comparing marketing automation modules for higher education admission. *In Computational Intelligence for Engineering and Management Applications: Select Proceedings of CIEMA, Springer Nature Singapore*, 885–912. https://doi.org/10.1007/978-981-19-8493-8_66

[6] Büyüközkan, G., Karabulut, Y., & Göcer, F. (2024). Spherical fuzzy sets based integrated DEMATEL, ANP, VIKOR approach and its application for renewable energy selection in Turkey. *Applied Soft Computing*, 158, 111465. https://doi.org/10.1016/j.asoc.2024.111465

[7] Cheng, S., Jianfu, S., Alrasheedi, M., Saeidi, P., Mishra, A. R., & Rani, P. (2021). A new extended VIKOR approach using q-rung orthopair fuzzy sets for sustainable enterprise risk management assessment in manufacturing small and medium-sized enterprises. *International Journal of Fuzzy Systems*, 23, 1347–1369. https://doi.org/10.1007/s40815-020-01024-3

[8] Chusi, T. N., Qian, S., Edalatpanah, S. A., Qiu, Y., Bayane Bouraima, M., & Ajayi, A. B. (2024). Interval-valued spherical fuzzy extension of SWARA for prioritizing strategies to unlock Africa's potential in the carbon credit market.*Computational Algorithms and Numerical Dimensions*, 3(3), 217–227. https://doi.org/10.22105/cand.2024.474739.1106

[9] Das, S., Roy, B. K., Kar, M. B., Kar, S., & Pamucar, D. (2020). Neutrosophic fuzzy set and its application in decision making. *Journal of Ambient Intelligence and Humanized Computing*, 11, 5017–5029. https://doi.org/10.1007/s12652-020-01808-3

[10] Du, W. S. (2019). Research on arithmetic operations over generalized orthopair fuzzy sets. *International Journal of Intelligent Systems*, 34(5), 709-732. https://doi.org/10.1002/int.22073

[11] Erdebilli, B., & Sicakyüz, C. (2024). An integrated q-rung orthopair fuzzy (q-ROF) for the selection of supply-chain management. *Sustainability*, 16(12), 4901. https://doi.org/10.3390/su16124901

[12] Fakhrehosseini, S. F. (2019). Selecting the optimal industrial investment by multi-criteria decision-making methods with emphasis on, TOPSIS, VIKOR, and COPRAS (case study of Guilan province).*International journal of research in industrial engineering*, 8(4), 312–324. https://doi.org/10.22105/riej.2020.216548.1117

[13] Farajpour Khanaposhtani, G. (2023). A new multi-attribute decision-making method for interval data using support vector machine.*Big Data and Computing Visions*, 3(4), 137–145. https://doi.org/10.22105/bdcv.2023.190406

[14] Hussain, Z., Abbas, S., & Yang, M. S. (2022). Distances and similarity measures of q-rung orthopair fuzzy sets based on the Hausdorff metric with the construction of orthopair fuzzy TODIM. *Symmetry*, 14(11), 2467. https://doi.org/10.3390/sym14112467

[15] Kiptum, C. K., Bouraima, M. B., Badi, I., Zonon, B. I. P., Ndiema, K. M., & Qiu, Y. (2023). Assessment of the challenges to urban sustainable development using an interval-valued Fermatean fuzzy approach. *Systemic analytics*, 1(1), 11–26. https://doi.org/10.31181/sa1120233

[16] Krishankumar, R., Nimmagadda, S. S., Rani, P., Mishra, A. R., Ravichandran, K. S., & Gandomi, A. H. (2021). Solving renewable energy source selection problems using a q-rung orthopair fuzzy-based integrated decision-making approach. *Journal of Cleaner Production*, 279, 123329. https://doi.org/10.1016/j.jclepro.2020.123329

[17] Kutlu Gündogdu, F., & Kahraman, C. (2019). A novel VIKOR method using spherical fuzzy sets and its application to warehouse site selection. *Journal of Intelligent & Fuzzy Systems*, 37(1), 1197-1211. 10.3233/JIFS-182651

[18] Liu, P., & Wang, P. (2018). Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. *International Journal of Intelligent Systems*, 33(2), 259–280. https://doi.org/10.1002/int.21927

[19] Liu, D., Peng, D., & Liu, Z. (2019). The distance measures between q-rung orthopair hesitant fuzzy sets and their application in multiple criteria decision making. *International Journal of Intelligent Systems*, 34(9), 2104–2121. https://doi.org/10.1002/int.22133

[20] Liu, Z. (2024). A distance measure of Fermatean fuzzy sets based on triangular divergence and its application in medical diagnosis. *Journal of Operations Intelligence*, 2(1), 167–178. https://doi.org/10.31181/jopi21202415

[21] Mahalakshmi, P., Vimala, J., Jeevitha, K., & Sri, S. N. (2024). Advancing cybersecurity strategies for multinational corporations: Novel distance measures in q-rung orthopair multi-fuzzy systems. *J. Oper. Strateg Anal*, 2(1), 49–55.

[22] Mahboob, A., Ullah, Z., Ovais, A., Rasheed, M. W., Edalatpanah, S. A., & Yasin, K. (2024). A MAGDM approach for evaluating the impact of artificial intelligence on education using 2-tuple linguistic q-rung orthopair fuzzy sets and Schweizer-Sklar weighted power average operator. *Frontiers in Artificial Intelligence*, 7, 1347626. https://doi.org/10.3389/frai.2024.1347626

[23] Mandal, U., Seikh, M.R. (2023). A novel score function-based EDAS method for the selection of a vacant post of a company with q-rung orthopair fuzzy data. *In Mathematics and Computer Science*, 1, 231–250. https://doi.org/10.1002/9781119879831.ch11

[24] Mishra, A. R., Chen, S. M., & Rani, P. (2022). Multiattribute decision making based on Fermatean hesitant fuzzy sets and modified VIKOR method. *Information Sciences*, 607, 1532–1549. https://doi.org/10.1016/j.ins.2022.06.037

[25] Nafei, A., Azizi, S. P., Edalatpanah, S. A., & Huang, C. Y. (2024). Smart TOPSIS: A neural Network-Driven TOPSIS with neutrosophic triplets for green Supplier selection in sustainable manufacturing. *Expert Systems with Applications*, 255, 124744. https://doi.org/10.1016/j.eswa.2024.124744

[26] Opricovic, S. (1998). Multicriteria optimization of civil engineering systems. *Faculty of civil engineering, Belgrade*, 2(1), 5-21.

[27] Palanikumar, M., Kausar, N., Ahmed, S. F., Edalatpanah, S. A., Ozbilge, E., & Bulut, A. (2023). New applications of various distance techniques to multi-criteria decision-making challenges for ranking vague sets. *Aims Mathematics*, 8(5), 11397–11424. 10.3934/math.2023577

[28] Pethaperumal, M., Jayakumar, V., Edalatpanah, S. A., Mohideen, A. B. K., & Annamalai, S. (2024). An enhanced MADM with L q* q-Rung orthopair multi-fuzzy soft set in healthcare supplier selection. *Journal of Intelligent & Fuzzy Systems*, (Preprint), 1–12. 10.3233/JIFS-219411

[29] Pinar, A., & Boran, F. E. (2020). A q-rung orthopair fuzzy multi-criteria group decision making method for supplier selection based on a novel distance measure. *International Journal of Machine Learning and Cybernetics*, 11, 1749–1780. https://doi.org/10.1007/s13042-020-01070-1

[30] Rani, P., & Mishra, A. R. (2020). Multi-criteria weighted aggregated sum product assessment framework for fuel technology selection using q-rung orthopair fuzzy sets. *Sustainable Production and Consumption*, 24, 90–104. https://doi.org/10.1016/j.spc.2020.06.015

[31] San Cristóbal, J. R. (2011). Multi-criteria decision-making in the selection of a renewable energy project in spain: The VIKOR method. *Renewable Energy*, 36(2), 498–502. https://doi.org/10.1016/j.renene.2010.07.031

[32] Seikh, M. R., & Mandal, U. (2022). Multiple attribute decision-making based on 3, 4-quasirung fuzzy sets. *Granular Computing*, 7, 965–978. https://doi.org/10.1007/s41066-021-00308-9

[33] Seikh, M. R., Mandal, U. (2022). Q-rung orthopair fuzzy Frank aggregation operators and its application in multiple attribute decision-making with unknown attribute weights. *Granular Computing*, 7, 709–730. https://doi.org/10.1007/s41066-021-00290-2

[34] Seikh, M. R., & Mandal, U. (2022). Multiple attribute group decision making based on quasirung orthopair fuzzy sets: Application to electric vehicle charging station site selection problem. *Engineering Applications of Artificial Intelligence*, 115, 105299. https://doi.org/10.1016/j.engappai.2022.105299

[35] Seikh, M.R., Mandal, U. (2023). Q-rung orthopair fuzzy Archimedean aggregation operators: Application in the site selection for software operating units. *Symmetry*, 15, 1680. https://doi.org/10.3390/sym15091680

[36] Torra, V. (2010). Hesitant fuzzy sets. *International Journal of Intelligent Systems*, 25(6), 529–539. https://doi.org/10.1002/int.20418

[37] Verma, R. (2020). Multiple attribute group decision-making based on order-a divergence and entropy measures under q-rung orthopair fuzzy environment. *International Journal of Intelligent Systems*, 35(4), 718–750. https://doi.org/10.1002/int.22223

[38] Verma, R., & Álvarez-Miranda, E. (2023). Group decision-making method based on advanced aggregation operators with entropy and divergence measures under 2-tuple linguistic Pythagorean fuzzy environment. *Expert Systems with Applications*, 231, 120584. https://doi.org/10.1016/j.eswa.2023.120584

[39] Wang, X., & Wang, K. (2021). A multi-criteria decision-making method based on triangular interval-valued fuzzy numbers and the VIKOR method. *Journal of Intelligent & Fuzzy Systems*, 40(1), 221–233. 10.3233/JIFS-191261

[40] Wang, D., Yuan, Y., Liu, Z., Zhu, S., & Sun, Z. (2024). Novel Distance Measures of q-Rung Orthopair Fuzzy Sets and Their Applications. *Symmetry*, 16(5), 574. https://doi.org/10.3390/sym16050574

[41] Yager, R. R. (2016). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 25(5), 1222–1230. 10.1109/TFUZZ.2016.2604005

[42] Yehudayoff, A. (2020). Pointer chasing via triangular discrimination. *Combinatorics, Probability and Computing*, 29(4), 485–494. https://doi.org/10.1017/S0963548320000085

[43] Yuan, J., Chen, Z., & Wu, M. (2023). A novel distance measure and CRADIS method in picture fuzzy environment. *International Journal of Computational Intelligence Systems*, 16(1), 186. https://doi.org/10.1007/s44196-023-00354-y

[44] Zare Ahmadabadi, H., Saffari Darberazi, A., Zamzam, F., Babakhanifard, M. S., Kiani, M., & Mofatehzadeh, E. (2024). A model of the factors affecting supply chain resilience: an integrative approach incorporating hesitant fuzzy TOPSIS and meta-synthesis. *Journal of Applied Research on Industrial Engineering*, 11(2), 195–211. https://doi.org/10.22105/jarie.2024.433396.1592